Number Theory

7th and 8th

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#GoBears

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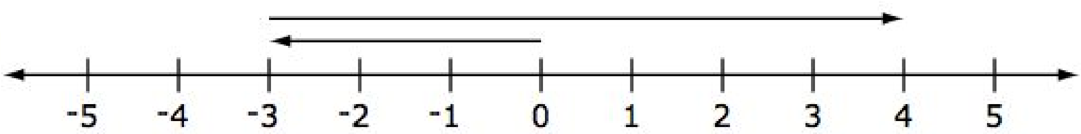
Description (Lesson Number):

(1)Students will be able to add and subtract positive and negative numbers multiply and divide positive and negative integers.
(2)Students will be able to correctly identify rational numbers including which of the two cards is greater. They will then be able to convert decimal representations into fractional representations.
(3)Students will apply prior knowledge of irrational numbers to estimate decimal values. Students will use their calculators to understand the implications of rounding numbers. This will be applied to a real life situation.
(4)In this lesson, students develop number sense through a series of three hands-on activities. Students explore the following concepts: the magnitude of a million, fractions between 0 and 1, and the effect of decimal operations.
(5)The objective of this lesson is to use combinations to solve KenKen puzzles. An early solution strategy is for students to guess and check and use logic-based elimination. This lesson builds on those strategies by having students systematically list all possible combinations within each cage, the darkly outlined sections of the puzzle.

Sample MCA Questions:

2. Which shows a model of $-3+4$?

A.



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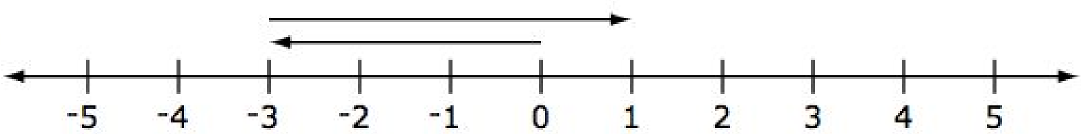

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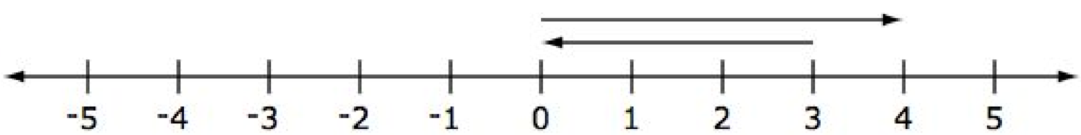

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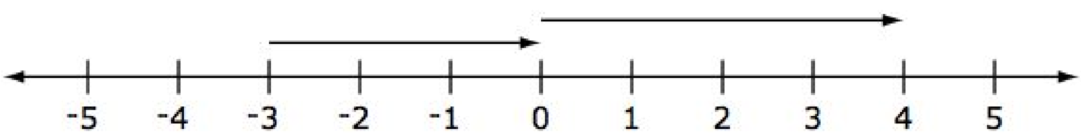

Part A Determine a decimal that is between $\frac{7}{9}$ and $\frac{4}{5}$. Show or explain your work.

Part B Determine a fraction that is between $0.\overline{63}$ and $\frac{2}{3}$. Show or explain your work.

Part C Order the numbers in the number set below from least to greatest. Show or explain your work.

Number Set				
$\frac{4}{5}$	$\frac{2}{3}$	$0.\overline{63}$	71%	$\frac{7}{9}$

Rainfall Reported (in inches)

City	Monday	Tuesday
Alexandria	$2\frac{3}{4}$	$1\frac{1}{8}$
Detroit Lakes	$2\frac{5}{16}$	$1\frac{1}{2}$
Park Rapids	$2\frac{3}{8}$	$1\frac{3}{8}$
Wadena	$2\frac{3}{16}$	$1\frac{3}{4}$

Four cities reported the amount of rainfall they received for 2 days. Which city's total amount of rainfall for the 2 days was the greatest?

A. Alexandria
B. Detroit Lakes
C. Park Rapids
D. Wadena

LESSON 1 :Chef's Hot and Cold Cubes (addition, subtraction, multiplication and division of integers) (3 - 4 days)

Adapted From Class Notes: MATH 6200, Bemidji State University Summer 2016
http://bfc.sfsu.edu/PRIME2/The-Chefs-Amazing-Soup.pdf

MN State Standards

Numbers and Operations

7.1.2.1	Add, subtract, multiply and divide positive and negative rational numbers that are integers, fractions and terminating decimals; use efficient and generalizable procedures, including standard algorithms; raise positive rational numbers to whole-number exponents.

Pre Test:

1) Add or Subtract the following:
   a) 4 + -3
   b) -4 + -3
   c) 4 - -3
   d) -4 - - 3
2) Multiply or Divide the following:
   a) $^{+}4 \bullet {}^{-}3$
   b) $^{-}6 \bullet {}^{-}4$
   c) $^{-}10 \div {}^{-}2$
   d) $^{+}24 \div {}^{-}8$

Part 1: Addition and Subtraction:

Objectives: Students will be able to add and subtract positive and negative numbers multiply and divide positive and negative integers.

LAUNCH: Ask students to come up with ways that they have seen negative numbers in their lives and how they may use them. Get into a discussion about the importance of negative and positive numbers and how important it is for them to be able to use them. Try to find a video that deals with negative and positive numbers.

Share the following story about the chef and hot and cold cubes:
The Chefs Amazing Soup (Adding and Subtracting Integers) In a far-of land, there was once a team of amazing chefs who cooked up the most wonderful soups ever imagined. They prepared their mixtures over a huge cauldron, and their work was very delicate and complex. During the cooking process, they frequently had to change the temperature of the cauldron in order to bring out certain flavors and cook the

soup to perfection. They adjusted the temperature of the soups either by adding special hot cubes or cold cubes to the cauldron or by removing some of the hot or cold cubes that were already in the cauldron. The cold cubes were similar to ice cubes except they didn't melt, and the hot cubes were similar to charcoal briquettes, except they didn't' lose their heat. If the number of hot cubes in the cauldron was the same as the number of cold cubes, the temperature of the cauldron was 0° on their temperature scale. For each hot cube that was put into the cauldron, the temperature went up one degree; for each hot cube removed, the temperature went down one degree. Similarly, each cold cube put in lowered the temperature one degree and each cold cube removed raised it one degree. The chefs used positive and negative numbers to keep track of the changes they were making to the temperature. For example, suppose 4 hot cubes and 10 cold cubes were dumped into the cauldron. Then the temperature would be lowered by 6° altogether, since 4 of the 10 cold cubes would balance out the 4 hot cubes, leaving 6 cold cubes to lower the temperature 6°, and they would write +4 + -10 = -6 to represent their actions and the overall final results of those actions. Similarly, if they added 3 hot cubes and then removed 2 cold cubes, the combined result would be to raise the temperature 5°. In that case they would write +3 - -2 = +5 And if they wrote -5 **-** +6 = -11, it would mean that first 5 cold cubes were added to the cauldron and then 6 hot cubes were removed, and that the combined result was to lower the temperature 11°

EXPLORE/SHARE: Put students into their groups and have them work through the following worksheet. Have students come up to the board and share/explain how they came up with their answers.

Directions: Show work for these problems on the board and have the students work in their groups showing work on another sheet of paper (if needed).

1. Each of the problems below describes an action taken by the chefs. Figure out how the temperature would change overall in each of these situations and write an equation to describe the action and the overall result.

a. Three cold cubes were added and five hot cubes were added.

b. Six hot cubes were added and eight cold cubes were added.

c. Nine cold cubes were added and two more cold cubes were added.

d. Ten cold cubes were added and six cold cubes were removed.

e. Five hot cubes were added and four cold cubes were removed.

f. Twelve hot cubes were added and seven cold cubes were removed.

Now work on these type of number problems in their groups having them share their answers on the board.
a. +23 + +5 =
b. +3 + -7 =
c. -4 + -10 =
d. -9 + +3 =
e. +5 - -1 =
f. +13 - +5 =

g. -8 - -5 =
h. -11 - +5 =
i. -4 + -8 =
j. -6 + 4 =
k. -9 – 5 =
l. -9 - -6 =
m. 10 + -6 =
n. 10 + 5 =

SUMMARIZE: Spend some time going over the importance of making sure that the students have the correct sign on their answers. You may want to have students try some individual problems and have volunteers come up to the board and explain their work.

Part 2: Multiply and Divide Positive and Negative Integers.

(Story and problems taken from this document, not sure who made it)
http://learn.shorelineschools.org/brookside/jjordan/documents/download/the_chefs_hot_and_cold_cubes12.doc?id=6938

Launch: Review the high points of the previous lesson on adding and subtracting integers. Begin now by reading the next portion of the chef story.

Chef Story Continued:
Sometimes they wanted to raise or lower the temperature by a large amount, but did not want to put the cubes into the cauldron one at a time. So for large jumps in temperature, they would put in or take out bunches of cubes.

For instance, if the chefs wanted to raise the temperature 100 degrees, then they might toss five bunches of 20 hot cubes each into the cauldron instead of 100 cubes one at a time. This saved a lot of time because they could have assistant chefs do the bunching.

When the chefs used bunches of cubes to change the temperature, they used a multiplication sign to record their activity. For example, to describe tossing five bunches of 20 hot cubes each into the cauldron, they would write

$$^{+}5 \cdot {}^{+}20 = {}^{+}100$$

where the $^{+}5$ meant that the five bunches were being added, and the $^{+}20$ showed that there were twenty hot cubes in each bunch.

The chefs could also change the temperature by removing bunches. For example, if they removed three bunches of 5 hot cubes each, the result was to lower the temperature 15 degrees, because each time a bunch of 5 hot cubes was removed, the temperature went down 5 degrees. To record this change, they would write

$$^{-}3 \cdot {}^{+}5 = {}^{-}15$$

where the $^{-}3$ meant that three bunches were being removed, and the $^{+}5$ showed that there were five hot cubes in each bunch.

Explore/Share: Have kids get in their groups and have them work together answering the following questions. Work at a pace that allows student groups to take turns sharing their answers up on the board.
(Questions taken from this document)
http://learn.shorelineschools.org/brookside/jjordan/documents/download/the_chefs_hot_and_cold_cubes12.doc?id=6938

1) Each of the problems below describes an action by the chefs. Figure out how the temperature would change overall in each of these situations and write an equation to describe the action and the overall result.

Hot and Cold Cubes Multiplication and Division

1. Five bunches of four cold cubes were removed. What was the change in temperature? Write an equation to describe the overall result.
2. Six bunches of eight hot cubes were added. What was the change in temperature? Write an equation to describe the overall result.
3. Three bunches of five cold cubes were removed. What was the change in temperature? Write an equation to describe the overall result.
4. Nine bunches of two cold cubes were added. What was the change in temperature? Write an equation to describe the overall result.
5. Seven bunches of four hot cubes were removed. What was the change in temperature? Write an equation to describe the overall result.
6. The temperature dropped eighteen degrees when nine bunches were added. How many cubes were in a bunch? Write an equation to describe the overall result.
7. The temperature dropped 48 degrees when six bunches were removed. How many cubes were in a bunch? Write an equation to describe the overall result.
8. The temperature rose 24 degrees when three bunches were removed. How many cubes were in a bunch? Write an equation to describe the overall result.
9. The temperature rose 25 degrees when five bunches were removed. How many cubes were in a bunch? Write an equation to describe the overall result.
10. The temperature dropped nine degrees when three bunches were added. How many cubes were in a bunch? Write an equation to describe the overall result.
11. The temperature rose 64 degrees when four bunches were removed. How many cubes were in a bunch? Write an equation to describe the overall result.

2) Describe the action involving hot or cold cubes that is represented by each if the following arithmetic expressions and state how the temperature would change overall.

a) $^{+}4 \cdot {}^{-}3$
b) $^{-}6 \cdot {}^{-}4$
c) $^{-}10 \div {}^{-}2$
d) $^{+}24 \div {}^{-}8$

Explain each problem in terms of the model of hot and cold cubes.

Your explanation should describe the action and state how the temperature changes overall.
1) $^{-}6 \cdot {}^{-}9$
2) $^{-}7 \cdot {}^{-}10$
3) $^{+}10 \div {}^{-}2$
4) $^{-}4 \cdot {}^{+}6$
5) $^{+}21 \div {}^{-}7$

6) $^-6 \cdot {}^+9$
7) $^-12 \div {}^-4$
8) $^+24 \bullet \div {}^-12$
9) $^-12 \bullet {}^+5$

Summarize:
Make sure students understand the difference between the sign differences when Adding and subtracting positive and negative numbers as compared to the rules the apply to multiplying and dividing positive and negative integers.

Post Test

3) Add or Subtract the following:
   a) 4 + -3
   b) -4 + -3
   c) 4 - -3
   d) -4 - - 3
4) Multiply or Divide the following:
   a) $^+4 \bullet {}^-3$
   b) $^-6 \bullet {}^-4$
   c) $^-10 \div {}^-2$
   d) $^+24 \div {}^-8$

Lesson 2: Decimal vs. Fraction War Game and Match Game (3 days)

Adapted from:
http://www.bemidjistate.edu/academics/departments/math_computer_science/smi/smi_archive/projects/2009/numbers/pdfs/Grade%207%208%20Steve%20Steve.pdf

MN State Standards

7.1.1.5	Recognize and generate equivalent representations of positive and negative rational numbers, including equivalent fractions. *For example:* $-\frac{40}{12} = -\frac{120}{36} = -\frac{10}{3} = -3.\bar{3}$.

Pre Test:

1) Convert $\frac{2}{5}$ to a decimal:
2) Convert .6 to a fraction:

Objectives: Students will be able to correctly identify rational numbers including which of the two cards is greater. They will then be able to convert decimal representations into fractional representations.

Launch: What are some of your favorite card games? My two favorite games are War and Memory. Let's play a game of War. Here is how we play for those who don't know/can't remember.

Explore: Split the class into pairs so that each game will have 2 players. Start with standard playing cards and play a game of War. Make sure pairings are based on those who know how to play and those who don't know how to play. When they become comfortable with the idea of playing a standard game, introduce the new card set of rational cards. The cards will have decimal and fractional representations. The set will have equivalent representations for both forms, (ex: .5 and ½ will both be in the set of cards.)

Begin Playing with the new "rational" cards.

Memory: Using the new set of rational cards, have the pairs now play memory. They need to arrange the cards in a rectangle depending on the size of the deck and have them face down. The goal is to match each decimal representation with its corresponding fractional representation.

Share: Do you notice anything about this game?
If we have a tie, how do we resolve that problem? Can you please show the class?
Once you play with the new cards, do you notice anything? Is this game harder? Why or Why not?

Is there anything you notice you can do to help speed up the process of the new game? What types of problems were more difficult?

Summarize: Review student's methods of converting fractions to decimals and decimals to fractions. Model several methods of doing the conversions.

Examples of Rational Cards:

$\frac{1}{2}$	$-\frac{1}{2}$	$4\frac{1}{6}$
4.1	.3333...	$-\frac{1}{3}$
$\frac{4}{7}$	$-\frac{2}{5}$	π
$-\pi$	3.14	$-\frac{22}{7}$
$\frac{3}{10}$	$-\frac{7}{3}$	$\frac{9}{2}$

$-\frac{4}{3}$	.9999...	$\frac{14}{5}$
$-\frac{15}{4}$	–3.9	.99
3	–4	–3.14
–.36	$-\frac{40}{12}$	$\sqrt{10}$
$-\sqrt{4}$	3.8	$-\sqrt{20}$
$-\frac{8}{9}$	–.8	–3

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$-\frac{1}{5}$	$-\frac{7}{2}$	$\frac{7}{2}$
$-\sqrt{16}$	$\sqrt{16}$	2.77
2.777...	$-2\frac{3}{10}$	–2.4
–5	5	0

Post Test:

1) Convert $\frac{2}{5}$ to a decimal:
2) Convert .6 to a fraction:

Lesson 3: Importance of Rounding (Bridge Problem) (1day)

Adapted From Class Notes: MATH 6200, Bemidji State University Summer 2016

MN State Standards

7.1.2.3	Understand that calculators and other computing technologies often truncate or round numbers. *For example*: A decimal that repeats or terminates after a large number of digits is truncated or rounded.
7.1.1.2	Understand that division of two integers will always result in a rational number. Use this information to interpret the decimal result of a division problem when using a calculator. *For example*: $\frac{125}{30}$ gives 4.16666667 on a calculator. This answer is not exact. The exact answer can be expressed as $4\frac{1}{6}$, which is the same as $4.1\overline{6}$. The calculator expression does not guarantee that the 6 is repeated, but that possibility should be anticipated.

Pre Test:

1) What will give you a more accurate number?
    a) 4 x 1.4 or 4 x $\sqrt{2}$

Objective:
Students will apply prior knowledge of irrational numbers to estimate decimal values. Students will use their calculators to understand the implications of rounding numbers. This will be applied to a real life situation.

Launch:
Ask students to share a story of their fears of walking over a bridge. What is the one question that will always come up? Will it hold me! How do the bridge builders know how much weight a bridge will hold.
Show the following the video and ask them what the driver is considering. (show only the first part of the video.
https://www.youtube.com/watch?v=x1Qgvv01XUU

Explore/Share:

In groups have students figure out the weight that the following bridge will hold. Make a chart to have students find the weight depending on the number of decimal places used.
Have students share their findings up on the board, discussing the implications of using more and less decimal places to come up with their total weight.

Equation to represent Weight Capacity of the Bridge: 10,000 lbs $(99-70 \cdot \sqrt{2})$

Answers:
1.4 = 10,000 lbs
1.41 = 3,000 lbs
1.414 = 200 lbs
1.4142 = 60 lbs
1.41421 = 57 lbs
$\sqrt{2}$ = 50.5 lbs

https://upload.wikimedia.org/wikipedia/commons/d/d4/Old_Bridge_ANF.JPG

Summarize:
Make sure students understand what is happening in the problem, why is the number getting smaller, why is it important to round. A discussion of when to round and when not to would also be appropriate here.. Have them come up with ideas of where rounding would be very important.

Post Test:
2) What will give you a more accurate number?
   a) 4×1.4 or $4 \times \sqrt{2}$

Lesson 4: Too Big or Too Small (3 days):

Adapted from Illuminations:
https://illuminations.nctm.org/TooBigTooSmall/

MN Math Standards:

7.1.1.1	Know that every rational number can be written as the ratio of two integers or as a terminating or repeating decimal. Recognize that π is not rational, but that it can be approximated by rational numbers such as $\frac{22}{7}$ and 3.14.
7.1.1.2	Understand that division of two integers will always result in a rational number. Use this information to interpret the decimal result of a division problem when using a calculator. *For example*: $\frac{125}{30}$ gives 4.16666667 on a calculator. This answer is not exact. The exact answer can be expressed as $4\frac{1}{6}$, which is the same as $4.1\overline{6}$. The calculator expression does not guarantee that the 6 is repeated, but that possibility should be anticipated.
8.1.1.1	Classify real numbers as rational or irrational. Know that when a square root of a positive integer is not an integer, then it is irrational. Know that the sum of a rational number and an irrational number is irrational, and the product of a non-zero rational number and an irrational number is irrational. *For example*: Classify the following numbers as whole numbers, integers, rational numbers, irrational numbers, recognizing that some numbers belong in more than one category: $\frac{6}{3}$, $\frac{3}{6}$, $3.\overline{6}$, $\frac{\pi}{2}$, $-\sqrt{4}$, $\sqrt{10}$, -6.7.
8.1.1.2	Compare real numbers; locate real numbers on a number line. Identify the square root of a positive integer as an integer, or if it is not an integer, locate it as a real number between two consecutive positive integers. *For example*: Put the following numbers in order from smallest to largest: 2, $\sqrt{3}$, -4, -6.8, $-\sqrt{37}$. *Another example*: $\sqrt{68}$ is an irrational number between 8 and 9.

Pre Test:

1) Explain how many fractions there are between the numbers 1 and 2?

2) Add 2.3 and 4.5.

Objective:
In this lesson, students develop number sense through a series of three hands-on activities. Students explore the following concepts: the magnitude of a million, fractions between 0 and 1, and the effect of decimal operations.

Activity 1: The size of a million dollars. (Day 1)

Launch: Ask students how much money they could fit into their pockets? Ask them to think about how big one million dollars is in one dollar bills. How much room do you think it would take to put one million dollars one dollar bills into a standard suitcase. If so, how large would the suitcase need to be? How heavy would it be? You may have students work in small groups (2 or 3 students per group) to explore these questions. (You may want to bring in a suitcase and fill it with reams of paper and figure out how many bills would be in a ream of paper.)

Explore/Share: In groups have students work through the following problems. After working through the problems have students share their answers on the board:

Begin the investigation by telling the following story:
Just as you decide to go to bed one night, the phone rings and a friend offers you a chance to be a millionaire. He tells you he won $2 million in a contest. The money was sent to him in two suitcases, each containing $1 million in one-dollar bills. He will give you one suitcase of money if your mom or dad will drive him to the airport to pick it up. Could your friend be telling you the truth? Can he make you a millionaire?
Involve students in formulating and exploring questions to investigate the truth of this claim. For example:

- Can $1,000,000 in one-dollar bills fit in a standard-sized suitcase? If not, what is the smallest denomination of bills you could use to fit the money in a suitcase?
- Could you lift the suitcase if it contained $1,000,000 in one-dollar bills? Estimate its weight.

Calculators should be available to facilitate and expedite the computation for analysis.
Note: The dimensions of a one-dollar bill are approximately 6 inches by 2.5 inches. Twenty one-dollar bills weigh approximately 0.7 ounces.
You may wish for students to locate these facts about dollar bills on their own, using internet or other appropriate resources. The students will also need to determine the dimensions of a "standard" suitcase.

Summarize: Make sure that all students understand how they came up with their answer. Focusing on the idea of how to solve the problem, pointing out that there may be more than one path to the answer.

Activity 2: Estimating Fractions Between 0 and 1, (Day 2)

Launch: List all the states between Minnesota and the the Gulf of Mexico, you may need a map. The list if finite, now have the students to list all the fractions between the numbers 0 and 1. Have students create a list and see how many they come up with. Is the list finite or infinite?

Explore/Share: Put students into groups and have them cut out the circles on the following handout. https://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/6-8/CircleTemplate.pdf

Have students put the circles together so that they can see the fractions printed on one side of one circle. Ask questions such as these:

- Show a small part of the shaded circle (less than ¼). Can you name the part represented?
- Show a large part of the shaded circle (greater than ¾). Can you name the part represented?

Ask students to reverse the circle with the printed fractions so that they cannot see the fractions. Ask students if they can:

- Show a fraction that is a little bigger than ½. What name can you give it?
- Show a fraction that is between ½ and ¾. What name can you give it?

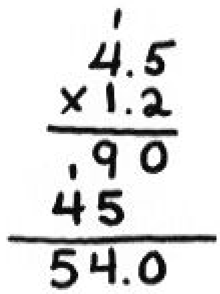
Continue asking questions that allow students to show their understanding of the fractions represented.
Other fraction models should also be used to evaluate students' understanding of fractions. During the course of your work have students share their responses to the questions asked making sure that all students understand what is being worked towards.

Summarize: Make sure that students understand the concept of the fractions between 0 and 1. This could also lead into a discussion on if it is possible to continue to find other fractions that were not discovered upon our activity.
Extension: Lead a discussion on converting fractions to decimal and percents.

Activity 3: Exploring The Effect of Operations on Decimals (Day 3)

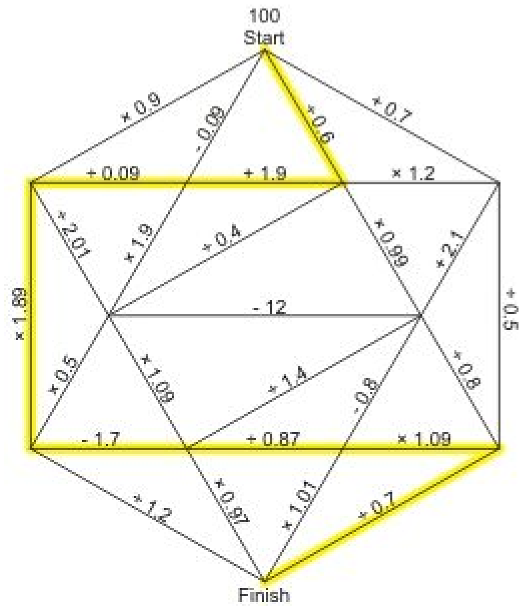
Launch: Have students add up the change in my pocket? How did they come up with the total? How about if you had dollars and cents? How did you come up with the total? Now put those problems on the board and ask them to add them.

Write the problem (as described next) on the chalkboard or overhead. Ask students to discuss what they notice. Lead a discussion that focuses on these key points:
In computing the product of 4.5 and 1.2, a student carefully lined up the decimals and then multiplied, bringing the decimal point straight down and reporting a product of 54.0.



Reflection on the answer should have caused the student to realize the product was too big. Multiplying 4.5 by a number slightly greater than 1 produces an answer a little more than 4.5. Instead, this student applied an incorrect procedure (line up the decimals in the factors and bring the decimal point straight down) and did not reflect on whether the resulting answer was reasonable.

EXPLORE / SHARE:
Tell students that they will be playing a game to practice decimal operations and their effects. Encourage students to trace several paths through the maze while always looking for the path that will yield the greatest increase in the calculator's display. (Note: Students often shy away from dividing by decimals less than 1, so you may want to discuss the general effect of dividing by a number less than 1 or multiplying by a number greater than 1. This could also be done as a pre-activity activity)
Give each student a calculator and a copy of the Maze Playing Board Activity Sheet
https://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/6-8/BigSmall-AS-Maze(1).pdf
Students are to choose a path through the maze. To begin, have the students enter 100 on their calculator. For each segment chosen on the maze, the students should key in the assigned operation and number. The goal is to choose a path that results in the largest value at the finish of the maze. Students may not retrace a path or move upward in the maze.
In pairs or in groups of three, students should discuss their strategies (after playing the game) and what worked best for them. Have them share with the rest of the class their discoveries and how they came up with the solutions.
(Students should be able to achieve a score in the thousands. The path highlighted below gives a result of roughly 6332.)



Extension: Possible follow-up activities include finding the path that leads to the smallest finish number or finding a path that leads to a finish number as near the start number (100) as possible.

Summarize: Make sure that students understand the importance of decimal addition/subtraction and multiplication and division. They should also be using their estimation skills to determine if their solutions are reasonable.

Post Test:

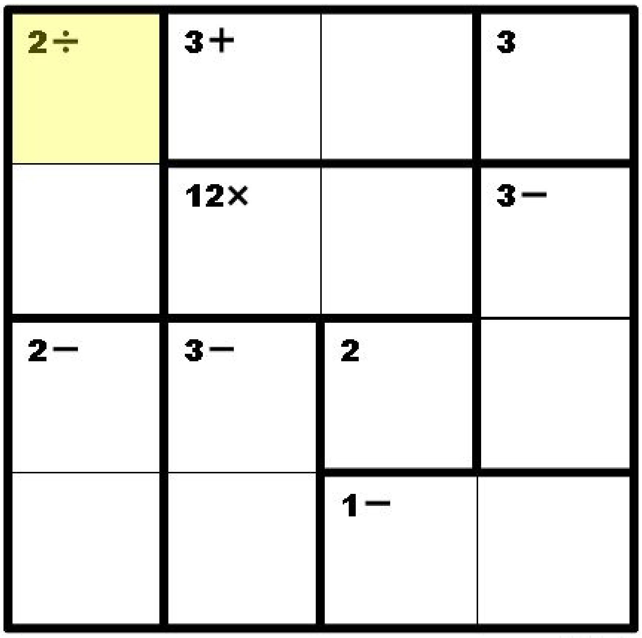
1) Explain how many fractions there are between the numbers 1 and 2?
2) Add 2.3 and 5.6.
3) If you had $1.25 and 75 cents, how much money do you have in total?

LESSON 5: Kan You Ken Ken (2-3 days)

Adapted from illuminations: https://illuminations.nctm.org/KenKenLP/
Key website: http://www.kenkenpuzzle.com/#

Objectives: The objective of this lesson is to use combinations to solve KenKen puzzles. An early solution strategy is for students to guess and check and use logic-based elimination. This lesson builds on those strategies by having students systematically list all possible combinations within each cage, the darkly outlined sections of the puzzle.

Pre Test: Have students try the 4x4 without any help as to what to do:



MN Math Standards:

7.1.2.1	Add, subtract, multiply and divide positive and negative rational numbers that are integers, fractions and terminating decimals; use efficient and generalizable procedures, including standard algorithms; raise positive rational numbers to whole-number exponents. *For example*: $3^4 \times \left(\frac{1}{2}\right)^2 = \frac{81}{4}$.

Activity 1: How to play KEN KEN

Launch:
What kind of puzzle you like to do?
What kind of puzzle do you think I like to do?
Being a math teacher I like math puzzles but I don't like hard ones like Sudoku. Here is a quick math puzzle to do.
You have 30 seconds, quickly using the numbers 1 - 4 once only, create the numbers 1 - 10. You can only use the same operation.

Explore/Share:

This lesson assumes that you and your students have played KenKen on an introductory level before. You may want to assess your students' understanding of the basic rules and terms before beginning a puzzle (ex: "What is a cage?"). If necessary, review the rules of KenKen with your students:

KenKen Rules

1. Every square in the grid will contain one number.
   - In a 3 x 3 puzzle, use the numbers 1–3.
   - In a 4 x 4 puzzle, use the numbers 1–4.
   - In a 5 x 5 puzzle, use the numbers 1–5.
   - In a 6 x 6 puzzle, use the numbers 1-6.
2. Do not repeat numbers in any row or column.
3. A *cage* is a heavily outlined set of squares. The numbers in the squares within a cage must combine (in any order) to produce the target number in the top corner. You must use the mathematical operation next to the target number.
4. Cages with just one square should be filled in with the target number in the top corner. These are called *freebies*, and this is a good place to start solving the puzzle.

As a teacher, try to resist helping students too much while they are solving their puzzles. They will learn and gain more by struggling to solve it on their own rather than receiving help from the teacher.
Now begin by having students work through a 3x3 puzzle and then a 4x4 puzzle.
(You can find different level of these puzzles here)
http://krazydad.com/inkies/

Have student present their different solutions to the puzzles on the board explaining their reasoning to solving the problems.

Summarize: Make sure student understand the process and rules of solving Ken Ken puzzle:

ACTIVITY 2: Going Deeper with Ken Ken:

Launch: Remember yesterday when I told you that I like math puzzles but only ones that I don't have to put a lot of thought into because at heart I am lazy, like most of you. Put up a 3x3 puzzle and have students fill out as fast as possible. Go over any questions students may have in solving. Show them a 5x5 puzzle and ask them how they would attack this puzzle. I like this but I want to do so that can have a few helps and actually solve it.

Explore/Share:
Have student in their groups, tell them, "Today we're going to explore some strategies for solving KenKen puzzles. Remember, we want to avoid guessing, so we will build our logic skills to find other ways of solving the puzzle. Today, we will focus on listing combinations and use inductive reasoning to find an appropriate arrangement of the combinations."
Distribute Puzzle 1 Activity Sheet. Review the rules for completing a KenKen puzzle as well as the vocabulary used (*cage, square, freebie*). This lesson should not be students' first activity with the puzzles so experience solving them is expected. The objective for this lesson is to be

explicit and thorough listing the possible combinations for each square, including all possible arrangements within the cage. The goal of the Puzzle 1 activity is for students to find ALL combinations and possible arrangements and develop strategies for systematically eliminating those that do not work.
Activity Sheet 1:
https://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/6-8/Can%20You%20KenKen%20AS%20Puzzle%201.pdf

Work through the worksheet having students share their answers to the questions.

Have students use the methods to work through a new 5x5 puzzle using some of the methods that you have been going over.

Summarization:
Review what you went over about solving larger puzzles making sure most of the students.

Activity 3: Working through Ken Ken with a plan

Launch: Ask kids to give their frustrations so far. Ask if any students have had success doing certain things when trying to find numbers to fit into the cages.

Explore/Share:
Puzzle #2 is designed to encourage students to reason about the solution process, make conjectures about the solution strategy, and defend their arguments. Have students work through activity #2 and share their work/solutions to the questions.
https://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/6-8/Can%20You%20KenKen%20AS%20Puzzle%202.pdf

Summarize:
Make sure students are becoming more comfortable with the process of solving a larger Ken Ken puzzle using methods other than guessing.

Activity 4: Mastering Ken Ken

Launch:
Name a game that you are really like, why do you like it because you are probably good at it. Today we are going to get good at Ken Ken.

Explore/Share:
In groups have students work in their groups and work on worksheet of the 5x5 and 6x6.

As the class progresses have students share their work on the board as they are solving the puzzles.
Activity:
https://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/6-8/Can%20You%20KenKen%20AS%20Puzzles%203And4.pdf

Summarize:
To wrap up the class, discuss the following questions together as a class. A good alternative is to have students answer these questions as a journal entry.

- What are some strategies you have noticed for limiting the number of combinations that may fit in a square?
- When is it helpful to list all possible combinations for a square?
- What is the biggest challenge in solving a KenKen puzzle? Explain.
- How did listing the combinations help you?
- How did listing the possible arrangements help you?
- How did your knowledge of factors and addends help you?
- What other strategies could you use to solve KenKen puzzles?

Extend: You can give kids much larger and difficult problems that could be done over a number of weeks as a project or extra credit.
The Ken Ken website has an interactive mode that allows students to play online.
http://www.kenkenpuzzle.com/

Post Test: Have students attempt the same 4x4 given in the pre-test